Analysis of Coupled Active Waveguides: Comparison of Different Modeling Techniques

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Abstract—**A self consistent model for the analysis of phase locked array lasers is proposed and a comparison is made with the more frequently used coupled mode theory. It is shown that in most cases the coupled mode solutions are in good agreement with the self-consistent solution, but in some cases care should be taken in applying the coupled mode theory and an improvement of that theory may be useful.**

I. Introduction

DURING the last years the research on multiple stripe (MS) lasers (or phase locked array laser) for high power, scanning [1]–[5] and single mode applications has increased significantly. MS lasers consist of a series of single stripe lasers, spaced closely enough so that the optical beams of the different lasers can interact. In this way these different beams show a certain phase relationship, which is characteristic for the supermode in which the MS laser operates. Depending on the exact structure, one of these supermodes may be favored with respect to the others and will be the first to reach threshold. These supermodes may be determined as the combination of the different modes of the individual lasers (the coupled mode theory, [6], [7]), or they may be obtained from the eigenmode analysis of the complete waveguide [8], [10]. In the next section a comparison will be made between both methods. It will be shown that in most cases the agreement between both methods is good. In special cases, however, the coupled mode theory needs to be used with some care.

The complex refractive index of the MS-laser waveguide is strongly dependent on the carrier density in the active layer. The carrier density itself is determined by the current injection into the active layer from the different contacts and by the diffusion of the carriers in the active layer itself. Hence an accurate analysis of this current spreading and of the drift and diffusion may be important in order to obtain a good representation of the effective refractive index [11]–[15]. Previously we have reported a self-consistent analysis in which the current distributions of the individual contacts were simply added [10], but this is only valid if the contacts are sufficiently separated. Recently we have discussed the influence of this current spreading and carrier diffusion more accurately for single stripe lasers [15]. In this paper we report the extension of that method to multiple stripe lasers and a comparison with the coupled mode theory is made. Next the coupled mode theory is improved to some extent and the influence of this improvement is discussed. In the following section the self-consistent model is briefly reviewed and the coupled mode analysis is discussed. In the last section, the examples are discussed and conclusions are made.

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II. Theory

The theory of the self-consistent model has been discussed in detail in [15]. The optical field for the ith mode is determined from the eigenmode equation:

$$\frac{d^2 Y_i}{dy^2} + \left(k_0^2 n^2(y) - \beta_{msi}^2\right) Y_i = 0 \tag{1}$$

in which $n(y)$ is determined from the effective refractive index method. The complex refractive index is linearly perturbed by the electron concentration, which itself is determined from a nonlinear diffusion equation taking into account the stimulated and spontaneous (bimolecular or monomolecular) recombination, as well as the degeneracy of the conduction and the valence band [15] and the current injection into the active layer. This current is determined from the solution of a two-dimensional potential problem in the top cladding layer [15]. The solution of (1) can either be found by means of a Beam Propagation Method (BPM) [9], [10], [15] or by means of an eigenvalue determination technique [8]. In the case of multilateral mode behavior (i.e., more than one mode is above threshold) it was found convenient to use a combination of both methods, although other techniques can be used [10].

The coupled mode theory is based on the following assumptions: 1) there is only nearest mode coupling, 2) the modes of the individual waveguides are orthogonal, 3) modes vanish in neighboring waveguides. In this paper we have dropped these assumptions and especially the influence of the last two assumptions will be discussed in the next section. The modes of the individual waveguides, denoted as G_k, are orthonormalized with a Gram–Schmidt orthogonalization procedure, from which a set of

K new function G'_k, (K is the number of stripes) is derived:

$$G''_k = \left[G_k - \sum_{j=1}^{k-1} \frac{\int G_k G'_j \, dy}{\int G'^2_j \, dy} G'_j \right], \quad k = 1, \cdots, K$$

$$G'_k = \frac{G''_k}{\sqrt{\int G''^2_k \, dy}}, \quad k = 1, \cdots, K. \tag{2}$$

The functions G'_k are orthogonal with respect to the product $\langle u, v \rangle$, defined as

$$\langle u, v \rangle = \int_{-\infty}^{+\infty} u(y) \, v(y) \, dy. \tag{3}$$

This orthogonality relation is also satisfied for the modes found from the eigenmode equation (1) of the MS waveguide. Note that other scalar products may be used in (2), such as

$$\langle u, v \rangle = \int_{-\infty}^{+\infty} u(y) \, v^*(y) \, dy. \tag{4}$$

It will be shown later that the orthonormalization procedure based on this scalar product yields poorer results. If we assume only nearest neighbor coupling, we can just keep the last term in the summation of (2) ($j = k - 1$). Assuming that Y_i can be written as

$$Y_i = \sum_{k=1}^{K} a^i_k G'_k \tag{5}$$

we can substitute (5) into (1). After multiplication with G'_1 and integration, we obtain a set of homogeneous equations in a^i_k, which only has a solution for specific values of β^2_{msi}. These values are considered to be good aproximations for the eigenvalues of (1), they are determined by

$$\det (M - \beta^2_{\text{msi}} E) = 0. \tag{6}$$

where

$$(M)_{ik} = k_0^2 \int G'_i G'_k n^2(y) \, dy + \int G'_i \frac{d^2 G'_k}{dy^2} \, dy.$$

Assuming only nearest mode coupling the only values of $(M)_{ik}$ which do not vanish are those for $k = i - 1, i, i + 1$. The second order derivatives appearing in (6) need not to be calculated numerically, since G'_k is a linear combination of the functions G_k, which satisfy a similar equation to (1). Furthermore we have $E_{ij} = \delta_{ij}$, with δ_{ij} the Kronecker symbol. It should be noted that (6) actually has the same form as the classical eigenvalue equation obtained from the coupled mode equations, but it does not assume that the modes G_k, are orthogonal, neither does it neglect the self-coupling, arising from the fact that the mode G_k has not completely vanished at the waveguides of the lasers $l \neq k$. Adopting these assumptions, (6) reduces to the classical coupled wave equations. Furthermore, it can easily be shown that the different supermodes found from (5) and (6) satisfy the orthogonality relation (3), since the matrix M is symmetric and the functions G'_k are orthogonal. If the classical coupled mode theory is used, one again needs to assume that the individual modes G_k are orthogonal in order to obtain the orthogonality (3) for the supermodes.

It should also be noted that one could use a variational principle to determine the eigenvalues of (1) [16]. Indeed, substituting (5) into the functional of [16]:

$$J = \frac{\int \left(\frac{d^2 Y_i}{dy^2} + k_0^2 n^2 Y_i \right) Y_i dy}{\int Y_i^2 dy} \tag{7}$$

and subsequently putting the derivatives of J, with respect to a^i_k, zero, we obtain (6) (Galerkins method). It should be remembered that the minimal value of J coincides with β^2_{msi}. Hence the value of β_{msi} obtained from (6) is the best estimation of the eigenvalue of (1) (according to the variational principle) which can be obtained with the repesentation of Y_i given by (5). As a consequence, the results obtained with the orthonormalization using G'^*_j in (2), may be expected to be worse.

III. Examples

Two examples will be discussed in some detail. First an example is discussed for which the coupled mode theory yields good results and the influence of the improvement given in the previous section is discussed. The method described in [10] is used to calculate the threshold field self-consistently and this serves as a reference solution for the coupled mode solutions. Next an example will be discussed for which the coupled mode theory (both the classical and the improved) fails to predict the laser array modes. For this example the self-consistent model described in the previous section is used to calculate the laser characteristics up to and beyond the onset of the second order lateral mode.

In the first example we consider the case of two index guided lasers, which slightly differ from one another (for example due to fabrication tolerances). The refractive index perturbation profiles of both lasers were chosen to be Gaussian, of 3-μm width (in $1/e$ points) and maximum values of 0.0029 and 0.003, respectively. The stripe widths were assumed to be 3 μm and the separation between the stripes was chosen to be 4 μm. For this laser we calculated the threshold field by means of the method of [10]. Next, we applied the coupled mode theory and the improved coupled mode theory, using the threshold fields and refractive indices of the individual lasers. The results for the even modes are shown in Fig. 1. The mode calculated with the self-consistent method is denoted as a and the mode calculated with the classical and improved coupled mode formalism are denoted as c and b, respec-

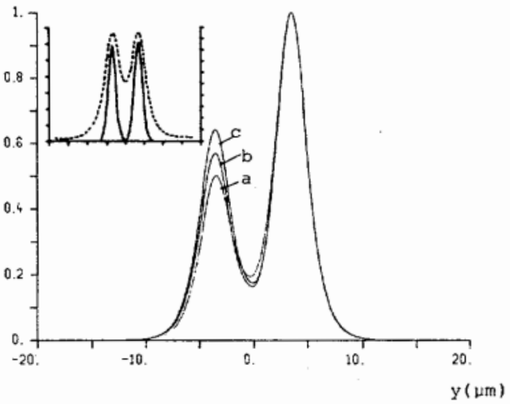

Fig. 1. Normalized field distributions (amplitude) for the multiple stripe laser, obtained with *a* the self-consistent analysis, *b* the improved coupled mode theory, and *c* the classical coupled mode theory. The inset shows the real (full line) and imaginary (dashed line) part of the refractive index.

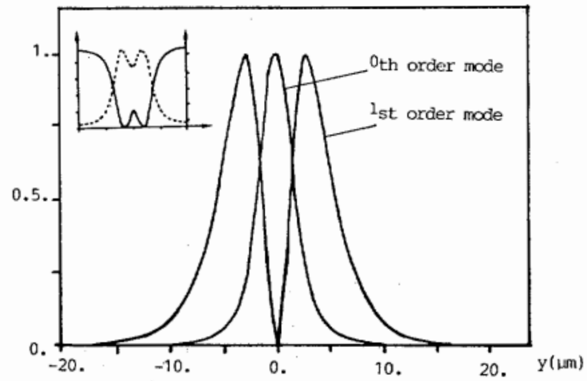

Fig. 2. Normalized intensity profiles of the fundamental (even) and the first order (odd) mode for the twin stripe laser of the second example. The inset shows the real and imaginary part of refractive index distribution along the lateral direction (varying from 3.489 up to 3.494 and from -1.8×10^{-3} up to 0.4×10^{-3}).

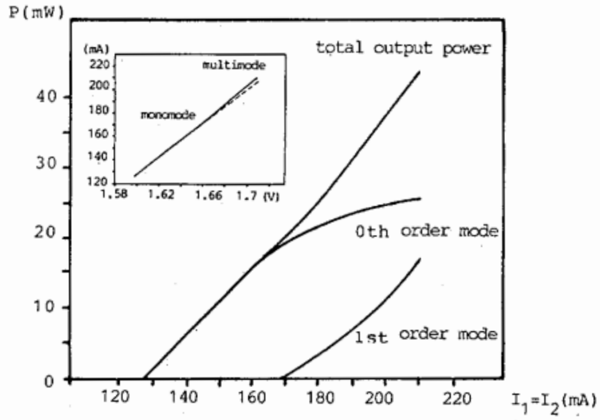

Fig. 3. Power-current characteristics for the fundamental and the first-order mode (see Fig. 2). The inset shows the current-voltage relationship for this device. Both in *I–V* and the *P–I* characteristic, a kink appears at the onset of the first order lateral mode.

tively. The improvement introduced by the coupled mode formalism can be seen from Fig. 1. The refractive index distribution for the laser array has been shown in the inset (real and imaginary part, varying from 3.475 to 3.478 and from -4×10^{-3} to 1×10^{-3}, respectively). Note that the antiguiding has been neglected. It was verified that the second orthonormalization procedure, discussed in the previous section yields poorer results. It should also be noted that the newly presented coupled mode formalism does not introduce any improvement for a perfectly symmetric twin stripe laser.

The second structure consists of two closely spaced, gain guided lasers (with a 5-μm contact each and separated by 2 μm). A single contact layer at constant potential was assumed. The sheet resistance of the top cladding layer was chosen to be 1000 $\Omega/\square$. The self-consistent method described in [15] was used to obtain the modes, the power current, and the current voltage characteristics. The inset in Fig. 2 shows again the refractive index (real and imaginary part). In a narrow region in between the contacts the local gain becomes slightly smaller than the required threshold gain. The refractive index however is slightly higher in between the contacts, due to antiguiding. As shown in Fig. 2, the first mode to reach threshold is a single lobed field occurring in between the contacts. Although the local gain is slightly too small there, the average gain (along the lateral direction) equals the required threshold gain. Such a solution cannot be found with the coupled mode formalism. The appearance of such a mode is always a consequence of the competition between the gain and index antiguiding. The antiguiding results in spatial regions with higher refractive index in between the contacts, focusing the light into it. If the loss in that region is not too high this mode may have a lower threshold current than the other modes. Similar results have been obtained in [10] for a three contact laser. Fig. 3 shows the power versus current characteristic for this device. A kink occurs at the onset of the first order lateral mode, as has already been reported in [8], [10] and as has also been observed experimentally. The inset shows the piecewise linear *I–V* curve, in which a kink appears at the onset of the first order lateral mode.

IV. Conclusion

We have compared an improved coupled mode formalism with a self-consistent analysis of phase-locked array lasers. The self-consistent method, which takes into account the interaction between the potential problem in the cladding layers and the diffusion (and stimulated emission) in the active layer, allows for the determination of both the *P–I* and *I–V* characteristics. Kinks in either the *P–I* or *I–V* curve indicate the onset of the second-order lateral mode. From the comparison between the self-consistent model and the coupled mode theory, it was shown that for most cases the coupled mode solutions are in good agreement with the self-consistent solution. For some cases however, especially those were the antiguiding becomes important, the coupled mode formalism was found to be inadequate. Furthermore, we presented an improved coupled mode formalism, which is slightly more accurate.

References

[1] A. Hardy and W. Streifer, "Coupled mode theory of parallel waveguides," *J. Lightwave Technol.*, vol. LT-3, no. 5, pp. 1135–1146, Oct. 1985.

[2] E. Marcatelli, "Improved coupled-mode equations for dielectric guides," *IEEE J. Quantum Electron.*, vol. 22, no. 6, pp. 988–993, June 1986.

[3] G. H. B. Thompson, "Compensation of line-broadening factor in twin stripe semiconductor lasers and improved modulation potential," *Electron. Lett.*, vol. 22, no. 12, pp. 621–622, 1986.

[4] W. K. Marshall and J. Katz, "Direct analysis of gain-guided phase-locked semiconductor lasers arrays," *IEEE J. Quantum Electron.*, vol. 22, no. 6, pp. 827–833, June 1986.

[5] F. Kappeler, "Dynamic single frequency low-chirp operation of a laterally coupled waveguide (LCW) laser," *Electron. Lett.*, vol. 20, no. 25/26, pp. 1040–1041, 1984.

[6] J. K. Butler, D. E. Ackley, and M. Ettenberg, "Coupled-mode analysis of gain and wavelength oscillation characteristics of diode laser phased arrays," *IEEE J. Quantum Electron.*, vol. QE-21, no. 5, pp. 458–464, May 1985.

[7] E. Kapon, J. Katz, and A. Yariv, "Supermode analysis of phase-locked arrays of semiconductor laser," *Opt. Lett.*, vol. 10, no. 4, pp. 125–127, Apr. 1984.

[8] J. Buus, "Models for the static and dynamic behavior of stripe geometry lasers," *IEEE J. Quantum Electron.*, vol. 19, no. 6, pp. 953–960, 1983.

[9] R. Baets, J. P. Van de Capelle, and P. E. Lagasse, "Longitudinal analysis of semiconductor lasers with low reflectivity facets," *IEEE J. Quantum Electron.*, vol. 21, no. 6, pp. 693–699, 1985.

[10] J. P. Van de Capelle, R. Baets, and P. E. Lagasse, "Self-consistent analysis of waveguiding in phase-locked array lasers," in *Integrated Optics*. New York: Springer Verlag, 1985, pp. 112–116.

[11] K. A. Shore and T. E. Rozzi, "Near-field control in multistripe geometry injection lasers," *IEEE J. Quantum. Electron.*, vol. 17, no. 5, pp. 718–722, May 1981.

[12] T. Kumar, R. F. Ormondroyd, and T. E. Rozzi, "Numerical solution of lateral current spreading and diffusion in near-threshold DH twin stripe lasers," *IEEE J. Quantum Electron.*, vol. 21, no. 5, pp. 421–433, May 1985.

[13] T. Kumar, R. F. Ormondroyd, and T. E. Rozzi, "A self-consistent model of the lateral behavior of a twin-stripe injection laser," *IEEE J. Quantum Electron.*, vol. 22, no. 10, pp. 1975–1985, 1986.

[14] M. C. Amann and F. Kappeler, "Analytical solution for the lateral current distribution in multiple stripe laser diodes," *Appl. Phys. Lett.*, vol. 48, no. 25, pp. 1710–1712, 1986.

[15] J. P. Van de Capelle, P. Vankwikelberge, and R. Baets, "Lateral current spreading in DH-lasers above threshold," *Proc. Inst. Elec. Eng.*, vol. 132, no. 2, pp. 143–148, 1986.

[16] D. P. Wilt and A. Yariv, "A self-consistent static model of the double-heterostructure laser," *IEEE J. Quantum Electron.*, vol. 17, no. 9, pp. 1941–1949, Sept. 1981.

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