Fast and Accurate Time-Domain Simulation of Passive Photonic Systems

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***Abstract*—This paper presents a novel approach for accurate and efficient time-domain simulations of general linear and passive photonic systems. Starting from the scattering parameters of the device or component under study, an equivalent baseband model in state-space form can be derived, which splits the optical carrier frequency and operates at baseband, thereby significantly reducing the modeling and simulation complexity without losing accuracy. The novel proposed method is validated via a suitable application example.**

***Index Terms*—Modeling and simulation, photonic integrated circuits, state-space representation, time-domain analysis.**

I. Introduction

Silicon photonics had a remarkable development in complexity and functionality in recent years, thanks to the progress in the manufacturing technology. Time-domain simulation is an essential part of the design flow of photonic integrated circuits, since it gives the most intuitive assessment of systems performance. Several time-domain simulation methods exist: finite-difference time-domain (FDTD), time-domain traveling wave (TDTW), split-step method (SSM), coupled mode theory (CMT), and convolution-based methods. However, for component- or circuit-level simulations, a trade off between accuracy and efficiency must be adopted when performing time-domain simulations via these techniques.

In this contribution, a novel approach for the analysis of general linear and passive photonic systems (such as waveguides, directional couplers, ring resonators) is proposed, which is based on the concept of lowpass equivalent signal and system representation defined in communication theory [1]. In particular, a suitable baseband model in state-space form is built for the lowpass equivalent representation of the system under study, which splits the optical carrier frequency and allows one to perform time-domain simulations at baseband efficiently. It is important to remark that the outputs of the photonic system under study can be analytically recovered from the outputs of the corresponding baseband model. The properties of the proposed methodology are discussed in details in the rest of the contribution.

II. Modeling and Simulation of Linear and Passive Photonic Systems

A. Equivalent Baseband Models Definition

The excitation signal of photonic systems is often an amplitude and/or phase modulated signal with *optical* carrier and *electronic* modulating signals, which can be described as

$$a(t) = A(t)cos(2\pi f_c t + \phi(t)) \tag{1}$$

where $A(t)$ and $\phi(t)$ are the time-varying amplitude and phase, respectively. An analytic complex-valued representation $a_a(t)$ of this real-valued modulated signal (1), called *analytic signal*, is introduced here as [1]

$$a_a(t) = a(t) + j\mathcal{H}(a(t)) = A(t)e^{j(2\pi f_c t+\phi(t))} \tag{2}$$

where $\mathcal{H}(a(t))$ is the Hilbert transform of $a(t)$. Now, the corresponding lowpass equivalent of the analytic signal can be defined as [1]

$$a_l(t) = a_a(t)e^{-j2\pi f_c} = A(t)e^{j\phi(t)}. \tag{3}$$

The relations between $a(t)$, $\mathcal{H}(a(t))$ and $a_l(t)$ are

$$a(t) = \mathrm{Re}(a_l(t)e^{j2\pi f_c t}) \tag{4}$$

$$\mathcal{H}(a(t)) = \mathrm{Im}(a_l(t)e^{j2\pi f_c t}). \tag{5}$$

Note that $a_l(t)$ is the complex envelope of $a(t)$ [1].

Let us assume that the linear and passive photonic system under study is described by a state-space model in the form

$$\begin{cases} \dfrac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = & \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{a}(t) \\ \boldsymbol{b}(t) = & \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{a}(t) \end{cases} \tag{6}$$

where $\mathbf{A} \in \mathbb{C}^{K\times K}$, $\mathbf{B} \in \mathbb{R}^{K\times N}$, $\mathbf{C} \in \mathbb{C}^{N\times K}$, $\mathbf{D} \in \mathbb{R}^{N\times N}$, K is the number of states in the state-vector $\boldsymbol{x}(t)$; N is the number of ports of the system under study and $\boldsymbol{a}(t)$ and $\boldsymbol{b}(t)$ are the vectors of the incident and reflected waves, respectively. The methodology to build this state-space model will be described in Section II-C. Now, the time-domain behavior of the system under study can be studied by solving the system of first-order ordinary differential equations (ODE) (6) for the given input signals $\boldsymbol{a}(t)$. However, a photonic system typically operates at frequencies around hundreds of terahertz, such as $[187;\ 200]$ THz, corresponding to a wavelength of $[1.5;\ 1.6]$ μm: a time-step of the order of femtoseconds is required to solve (6), given that such time-step must be smaller than the period of the highest frequency component of the signals considered. Hence, time-domain simulations based on (6) can be computationally expensive and memory consuming. In order to address this

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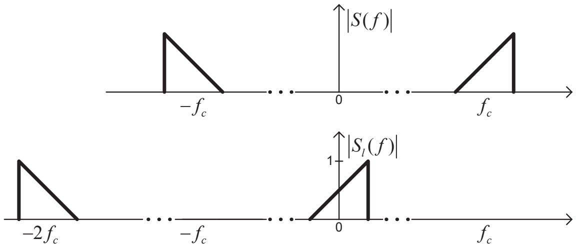

Fig. 1. Relation between the frequency response of the physical state-space model (6) (top) and of the new derived baseband state-space model (8) (bottom).

issue, an equivalent baseband model in state-space form is derived in the following.

By applying the Hilbert transform to (6) and by expressing $\boldsymbol{a}(t)$, $\boldsymbol{b}(t)$ and $\boldsymbol{x}(t)$ in the forms (4) and (5), leads to

$$\begin{cases} \dfrac{\mathrm{d}\left(\boldsymbol{x_l}(t)e^{j2\pi f_c t}\right)}{\mathrm{d}t} = & \boldsymbol{A}\boldsymbol{x_l}(t)e^{j2\pi f_c t} + \boldsymbol{B}\boldsymbol{a_l}(t)e^{j2\pi f_c t} \\ \boldsymbol{b_l}(t)e^{j2\pi f_c t} = & \boldsymbol{C}\boldsymbol{x_l}(t)e^{j2\pi f_c t} + \boldsymbol{D}\boldsymbol{a_l}(t)e^{j2\pi f_c t}. \end{cases} \tag{7}$$

After simple mathematical manipulations, (7) can be written as

$$\begin{cases} \dfrac{\mathrm{d}\boldsymbol{x_l}(t)}{\mathrm{d}t} = & (\boldsymbol{A} - j2\pi f_c \boldsymbol{I})\boldsymbol{x_l}(t) + \boldsymbol{B}\boldsymbol{a_l}(t) \\ \boldsymbol{b_l}(t) = & \boldsymbol{C}\boldsymbol{x_l}(t) + \boldsymbol{D}\boldsymbol{a_l}(t) \end{cases} \tag{8}$$

where $\boldsymbol{I}$ is the identity matrix. We define (8) as the *equivalent baseband (state-space) model* of the photonic system represented by (6). In frequency domain, it can be proven that the frequency response $\boldsymbol{S}_l(f)$ of the model (8) is equivalent to the frequency response $\boldsymbol{S}(f)$ of the model (6) shifted by f_c [2], as illustrated in Fig. 1.

Now, the time-domain simulation of the photonic system considered can be performed in baseband by solving (8) with respect to the lowpass input signal (3): given that the frequency spectrum of the lowpass signals is of the order of the gigahertz, since it depends only on the electronic modulating signal as described by (1) and (3), a relatively large time-step can be used. Finally, the output of the photonic system can be analytically recovered from the lowpass equivalent output, see (4). The relations among photonic signals and systems and their equivalent counterparts are shown in Fig. 2.

B. Passivity of Equivalent Baseband Models

It is of paramount importance for time-domain simulations that relevant physical properties of the system under study, namely stability and passivity, are guaranteed [3]. In the following, it will be investigated if the proposed equivalent baseband models still preserve such properties.

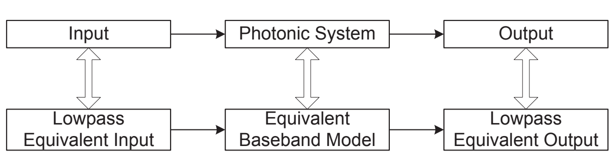

Fig. 2. Time-domain simulation of equivalent baseband models.

According to [4], [5], an n-port electronic system is passive if, for any $\tau > -\infty$ and $\boldsymbol{v}(t) \in L_{2n}$ (L_{2n} denotes the space of all vectors whose n components are functions of a real variable t and square integrable over $-\infty < t < \infty$), it holds

$$\mathrm{Re} \int_{-\infty}^{\tau} \boldsymbol{v}^H(t)\boldsymbol{i}(t)dt \geq 0 \tag{9}$$

where $\boldsymbol{v}(t)$, $\boldsymbol{i}(t)$ are the voltage and current at the system ports. It is important to note that this definition is given not only for real signals but also for complex ones. By expressing (9) in terms of the forward $\boldsymbol{a}(t)$ and backward $\boldsymbol{b}(t)$ waves, the passivity definition becomes [4], [6]

$$\int_{-\infty}^{\tau} \boldsymbol{a}^H(t)\boldsymbol{a}(t) - \boldsymbol{b}^H(t)\boldsymbol{b}(t)dt \geq 0 \tag{10}$$

which is more convenient to describe photonic systems. Note that the superscript ^{H} stands for the transpose conjugate operator.

Following the same proof process in [4], particularly *Theorem 2* and *Theorem 3*, the following passivity constraints on the scattering parameters $\boldsymbol{S}_l(s)$ of the equivalent baseband models can be derived from (10) [2]:

1) $\boldsymbol{S}_l(s)$ is analytic in $\mathrm{Re}(s) > 0$;
2) $\boldsymbol{I} - \boldsymbol{S}_l^H(s)\boldsymbol{S}_l(s)$ is a nonnegative-definite matrix for all s such that $\mathrm{Re}(s) > 0$.

The first condition is related to causality and stability; while the second one basically ensures that $\boldsymbol{S}_l(s)$ is bounded. Alternatively, the same conclusions can be reached via the approach in *Chapter II* of [6], which gives a simpler formal proofs by using the theory of distributions. The interested reader may consult [4] and [6] for a detailed and comprehensive proof. Note that the passivity constraints on the scattering parameters of physical systems are identical, but one additional condition must be satisfied: $\boldsymbol{S}(s^*) = \boldsymbol{S}^*(s)$. This constraint ensures that the system impulse response is real [7], so that a real input results in a real output, and makes the system physically realizable [4], [6]: this is not necessary for the equivalent baseband models presented in this contribution, since they are defined as a mathematical representation of the system under study with complex-valued input and output signals.

Hence, the passivity of baseband models in the form (8) can be assessed by the same methods applicable to state-space models in the form (6). For example, the Hamiltonian matrix $\boldsymbol{M}_l$ for the equivalent baseband models can be derived by following the same process in [8], leading to

$$\boldsymbol{M}_l = \begin{bmatrix} \boldsymbol{A}_l - \boldsymbol{B}\boldsymbol{L}^{-1}\boldsymbol{D}^H\boldsymbol{C} & -\boldsymbol{B}\boldsymbol{L}^{-1}\boldsymbol{B}^H \\ \boldsymbol{C}^H\boldsymbol{Q}^{-1}\boldsymbol{C} & -\boldsymbol{A}_l^H + \boldsymbol{C}^H\boldsymbol{D}\boldsymbol{L}^{-1}\boldsymbol{B}^H \end{bmatrix} \tag{11}$$

where

$$\boldsymbol{A}_l = \boldsymbol{A} - j2\pi f_c \boldsymbol{I}$$
$$\boldsymbol{L} = \boldsymbol{D}^H\boldsymbol{D} - \boldsymbol{I}$$
$$\boldsymbol{Q} = \boldsymbol{D}\boldsymbol{D}^H - \boldsymbol{I}.$$

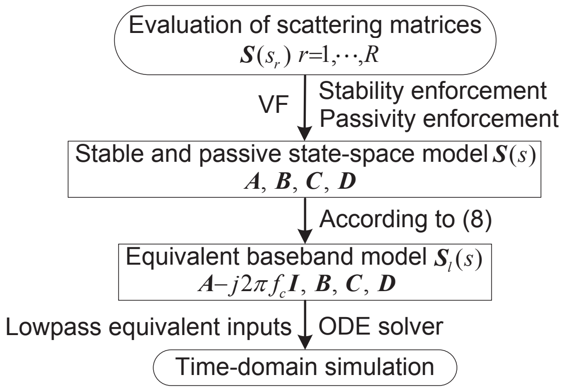

Fig. 3. Flow chart of the proposed modeling strategy.

C. Proposed Modeling Framework

Starting with the scattering parameters of the system under study evaluated for a set of frequency samples around the carrier frequency, the Vector Fitting (VF) algorithm [9]–[12] can be adopted to compute a stable and passive pole-residue model in the form

$$\boldsymbol{S}(s) = \sum_{q=1}^{Q} \frac{\boldsymbol{R}_q}{s - p_q} + \boldsymbol{D} \tag{12}$$

where the poles p_q and residue matrices $\boldsymbol{R}_q \in \mathbb{C}^{N\times N}$ are either real or complex conjugate pairs, the matrix term $\boldsymbol{D} \in \mathbb{R}^{N\times N}$ is the same as in (6), Q is the total number of the poles p_q and residue matrices $\boldsymbol{R}_q$. A pole-flipping scheme is used to enforce stability [9], while passivity assessment and enforcement can be accomplished using the robust standard techniques [8], [11], [12]. Then, such pole-residue model can be readily converted into a stable and passive state-space representation (6) [8]: the matrix $\boldsymbol{A}$ can be expressed as a diagonal matrix containing all the poles p_q, while $\boldsymbol{C}$ is formed by all the residues $\boldsymbol{R}_q$, and $\boldsymbol{B}$ contains only elements equal to zero or one. Hence, the desired equivalent baseband model (8) can be obtained by shifting all the diagonal elements in $\boldsymbol{A}$ by $j2\pi f_c$ and its passivity can be checked by means of the Hamiltonian matrix (11). The flow chart of the proposed modeling strategy is shown in Fig. 3.

In [1], [13] a similar poles-shifting approach for microwave systems is directly carried out only on the pole-residue model in the Laplace domain. Then, the time-domain simulation in both [1] and [13] is performed via convolution. In this paper, equivalent baseband models are defined as a linear, time-invariant, continuous systems whose time-domain simulation is conducted by directly solving the ODE in (8). Furthermore, the passivity conditions on both the equivalent baseband models and corresponding scattering parameters are derived.

III. Numerical Example

The cascaded-rings bandpass filter in [14] is studied in this section and its structure is shown in Fig. 4. Five rings with radius of 477 μm are cascaded to form a periodic bandpass filter having a bandwidth of 20 GHz and a free spectral range of 100 GHz, which is designed for wavelength division multiplexing systems.

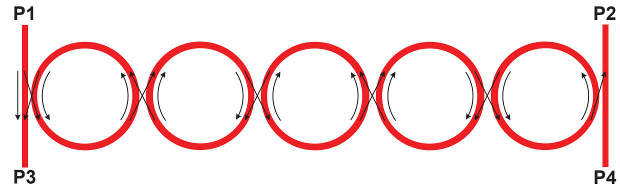

Fig. 4. Cascaded-ring-resonators bandpass filter.

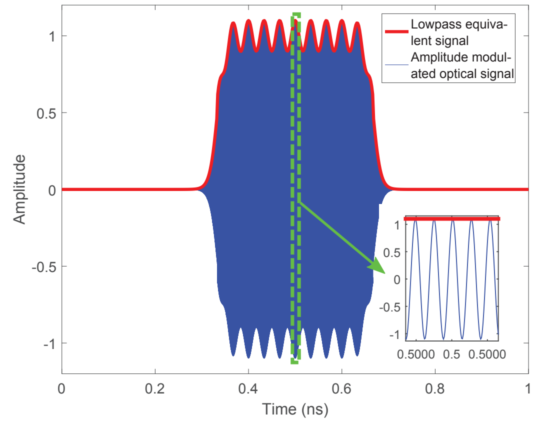

Fig. 5. The amplitude modulated optical signal (blue line) and the corresponding lowpass equivalent signal (red line).

Assuming an electronic pulse signal (width of 0.4 ns) with a sinusoidal noise of frequency 30 GHz as modulating signal, the filter is used to eliminate the noise. First, this pulse signal is modulated over an optical carrier with frequency 190.57 THz. According to equation (3), the corresponding lowpass equivalent signal is the pulse itself, since only amplitude modulation is considered. These two signals are illustrated in Fig. 5. Then, starting from the scattering parameters of the filter simulated in the frequency range of $\pm$50 GHz around the carrier frequency in the optical circuit simulator *Caphe*[1], a suitable stable and passive state-space model (6) is built via the VF algorithm by using 14 poles, with maximum absolute error of less than -60 dB. The corresponding equivalent baseband state-space model can be directly derived according to (8). Figure 6 shows the frequency-domain accuracy of the computed baseband model.

Assuming that port P1 is excited with the input signal, the time-domain simulations of these two models with their input signals are conducted in Matlab[2] with time-step 0.22 fs and 3.3 ps, respectively. The output signals of these two models are shown in Fig. 7. As expected, the 30 GHz sinusoidal noise is removed from the input signal, and the absolute value of the complex output from the equivalent baseband model (8) is

[1]Part of the IPKISS design suite: http://www.lucedaphotonics.com

[2]The Mathworks Inc., Natick, MA, USA

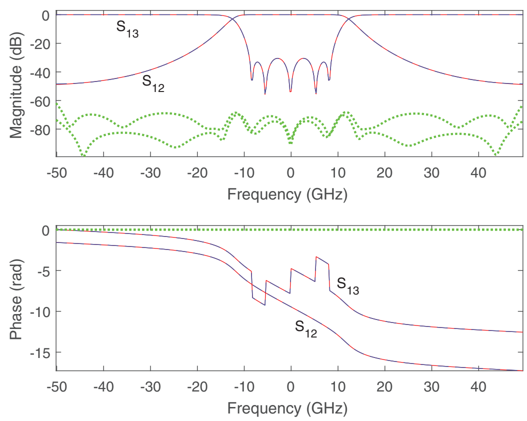

Fig. 6. Comparison of the magnitude (top) and phase (bottom) of the filter scattering parameters extracted via Caphe (full blue line) and the built baseband model (red dashed line), where the green dots represent the corresponding absolute error.

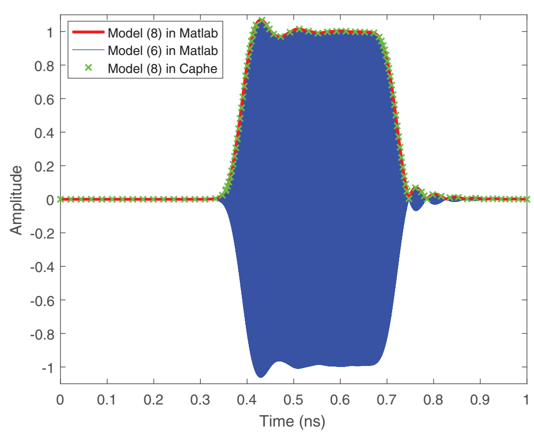

Fig. 7. The output at port P2 of the filter, the red line and green $\times$ are the absolute value of the complex output of the baseband model (8) simulated in Matlab and Caphe, respectively, while the blue line is the output obtained with the state-space model (6) simulated in Matlab.

exactly the envelope of the output of the state-space model (6). The time-domain simulations of model (6) and (8) in Matlab require 42 s and 0.01 s, respectively, which demonstrate the efficiency of the proposed technique. Furthermore, the baseband model (8) can also be readily implemented and simulated in the commercial tool *Caphe*, giving consistent results as shown in Fig. 7.

IV. Conclusion

A novel time-domain modeling and simulation method for the accurate and efficient analysis of general linear and passive photonic systems is described in this paper. The proposed technique is applicable to a large range of photonic devices and components (such as waveguides, directional couplers, ring resonators, etc.), since it is based on the scattering parameters representation. In particular, an equivalent baseband model in state-space form of the system under study is derived, which allows one to efficiently perform time-domain simulations at baseband. The desired model is obtained via the robust VF algorithm, which ensures the preservation of fundamental properties for time-domain simulations, such as stability and passivity.

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